15.5 Electric Power and Energy

SECTION OUTCOMES

- Explain the concept and related units of electric energy.
- Compare and describe the concepts of electric energy and electric power.

KEY TERM

- kilowatt-hour

When a solar storm sent an unexpectedly high flow of electrons into Earth's magnetic field, the energy surge caused an overload of the power grid serving the northeastern United States. An area of more than 200 000 square kilometres was blacked out, including the cities of New York, Buffalo, and Boston. People were trapped in elevators and on subways; hospitals, including operating rooms, were plunged into darkness.

Figure 15.25 Moonlight reflects from the darkened windows of New York City buildings during a massive power failure in 1965, which affected a large part of both Canada and the United States. As well as major inconveniences, riots and looting marked the event, which highlighted society's dependence on electric energy.

A major power failure is not required to remind us of how dependent we are on electric energy. Even a brief interruption of the electric energy supply demonstrates how reliant we have become on electricity. A major reason why electricity has become such a dominant energy form is the ease with which it can be transmitted and with which it can be converted into other forms of energy.

Power Output, Potential Difference, Current, and Resistance

Every appliance is rated for its power output (P)—the rate at which it can transform electric energy to a desired form (light, sound, or heat, for example). At the high end of the scale, an electric range might have a power rating greater than 12 000 W. An electric clothes dryer might be rated at 5000 W. At the other end of the scale, an electric shaver might be rated at 15 W and a clock at only 5 W.

In Chapter 6, you learned that power is defined as work done per unit time or energy transformed or transferred per unit time.

$$P = \frac{W}{\Delta t} \qquad \text{Power} = \frac{\text{work done}}{\text{time interval}}$$

$$P = \frac{\Delta E}{\Delta t} \qquad \text{Power} = \frac{\text{energy transformed}}{\text{time interval}}$$

The definition is the same whether you are referring to mechanical energy or electric energy. In a circuit, the loads transform electric potential energy into heat, light, motion, or other forms of energy. In power lines, electric energy is transferred from one location to another.

When working with electric circuits and systems, you typically work with potential difference, current, and resistance. It would be convenient to have relationships among power and these commonly used variables. To develop such relationships, start with the definition of potential difference.

- Definition of electric potential difference $\qquad V = \dfrac{\Delta E_Q}{q}$
- Solve for electric potential energy. $\qquad Vq = \dfrac{\Delta E_Q}{\cancel{q}}\cancel{q}$

  $\Delta E_Q = qV$
- Recall the definition of power. $\qquad P = \dfrac{\Delta E}{\Delta t}$
- Substitute the expression for electric potential energy into the equation for power. $\qquad P = \dfrac{qV}{\Delta t}$
- Recall the definition of current. $\qquad I = \dfrac{q}{\Delta t}$
- Substitute I for $\frac{q}{\Delta t}$ in the equation for power. $\qquad P = IV$

Figure 15.26 The power rating (sometimes called the "wattage") of a light bulb tells you how fast it will convert electric energy into heat and light. For an incandescent bulb, only about 2 percent of the transformed energy is actually emitted as light; the rest is emitted as heat. A fluorescent bulb, on the other hand, converts about 9.5 percent of its energy into light, making it more than four times as efficient as an incandescent bulb.

ELECTRIC POWER

Power is the product of current and potential difference.

$$P = IV$$

Quantity	Symbol	SI unit
power	P	W (watt)
current	I	A (ampere)
potential difference	V	V (volt)

Unit Analysis

(power) = (current)(potential difference) $\mathrm{W} = \mathrm{A} \cdot \mathrm{V}$

To verify that a watt is equivalent to an ampere times a volt, recall that an ampere is defined as a coulomb per second. $\mathrm{A} = \frac{\mathrm{C}}{\mathrm{s}}$

Recall that a volt is defined as a joule per coulomb. $\mathrm{V} = \frac{\mathrm{J}}{\mathrm{C}}$

Substitute these units into W = A · V.

$$\mathrm{W} = \mathrm{A} \cdot \mathrm{C}$$

$$\mathrm{W} = \left(\frac{\cancel{\mathrm{C}}}{\mathrm{s}}\right)\left(\frac{\mathrm{J}}{\cancel{\mathrm{C}}}\right)$$

When you cancel units (coulomb), you have shown that a watt is a joule per second, as defined on page 262. $\mathrm{W} = \frac{\mathrm{J}}{\mathrm{s}}$

MODEL PROBLEM

Calculating Electric Power

What is the power rating of a segment of Nichrome™ wire that draws a current of 2.5 A when connected to a 12 V battery?

Frame the Problem

- The *power rating* of an object is the *rate* at which it *converts electric energy* into another form of energy.
- When *current* passes through Nichrome™ wire, the resistance of the wire causes some energy to be converted into heat.
- You can calculate *power* from the *current* and *potential difference* across the ends of the wire.

Identify the Goal

The power rating, P, of the wire

Variables and Constants

Known	Unknown
$I = 2.5$ A	P
$V = 12$ V	

Strategy	Calculations
Use the equation that relates current and potential difference to power.	$P = IV$
Substitute the known values.	$P = (2.5\ \text{A})(12\ \text{V})$
An A · V is equivalent to a W.	$P = 30\ \text{A} \cdot \text{V}$
	$P = 30\ \text{W}$
The power rating of the segment of wire is 30 W.	

Validate

The units combine to give watts, which is correct for power.

Refer to the derivation on page 735 to review why current times potential difference is a correct expression for power.

PRACTICE PROBLEMS

40. An electric toaster is rated at 875 W at 120 V.

- **(a)** Calculate the current the toaster draws when it is on.
- **(b)** Calculate the electric resistance of the toaster.

41. A light bulb designed for use with a 120 V power supply has a filament with a resistance of 240 Ω.

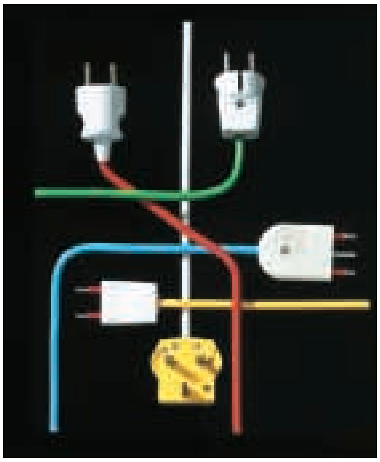
- **(a)** What is the power output of the bulb when the potential difference is 120 V?
- **(b)** If the bulb is inadvertently connected to an 80.0 V power supply, what would be the power output of the bulb?
- **(c)** If you wanted to construct a bulb to use with an 80.0 V power supply so that it would have the same power output as a 240 Ω bulb connected to a 120 V power supply, what should be the resistance of the bulb's filament?

42. A heater has a resistance of 15 Ω.

- **(a)** If the heater is drawing a current of 7.5 A, what is its power output?
- **(b)** If the current to the heater was cut in half, what would happen to the power output?

Technology Link

Different appliances, especially those that have large power outputs, often have a specially designed plug shape that is legal only for that type of appliance. Why is this necessary?



If you read the power rating on any appliance, you will notice that it is always accompanied by the potential difference that produces the specified power output. In Canada and the United States, electric energy is provided at a potential difference of 120 V, while in many other countries (including most of Europe) electric energy is provided at a potential difference of 240 V. Travellers taking appliances such as hair dryers to Europe find that these appliances are often damaged by the higher potential difference of the power supply.

Figure 15.27 Some small hair dryers designed for travel have switches that allow you to select the voltage rating. The dryer will provide the specified power output and not be damaged, whether you plug it into a 120 V line or a 240 V line.

MODEL PROBLEM

Using the Wrong Power Source

In North America, the standard electric outlet has a potential difference of 120 V. In Europe, it is 240 V. How does the dissimilarity in potential difference affect power output? What would be the power output of a 100 W–120 V light bulb if it was connected to a 240 V system?

Frame the Problem

- A light bulb contains a fine filament that has a certain electric *resistance*.
- The specifications (100 W–120 V) on the bulb mean that when it is connected to a 120 V potential difference, it will convert electric energy into light (and thermal energy) at 100 W.
- The resistance will remain approximately the same, even if it is connected to a different potential difference. Therefore, according to *Ohm's law*, the *current* will be different.
- If both the *current* and *potential difference* have *changed*, the *power* will likely be different.
- The key quantity is the *resistance*.

Identify the Goal

The power output, P, when the light bulb is connected to a 240 V line

Variables and Constants

Known	**Unknown**
$V_{120} = 120$ V	P_{240}
$P_{120} = 100$ W	I_{240}
$V_{240} = 240$ V	I_{120}
	R

(**Note:** Subscripts refer to the potential difference of the line to which the light bulb is connected.)

Strategy

Since there is no direct method to calculate the power, several steps will be involved. A tree diagram will help you determine what steps to take.

You can find the power output at 240 V if you know the current and potential difference at 240 V.

The potential difference is known.

You can find the current at 240 V if you know the potential difference at 240 V and the resistance.

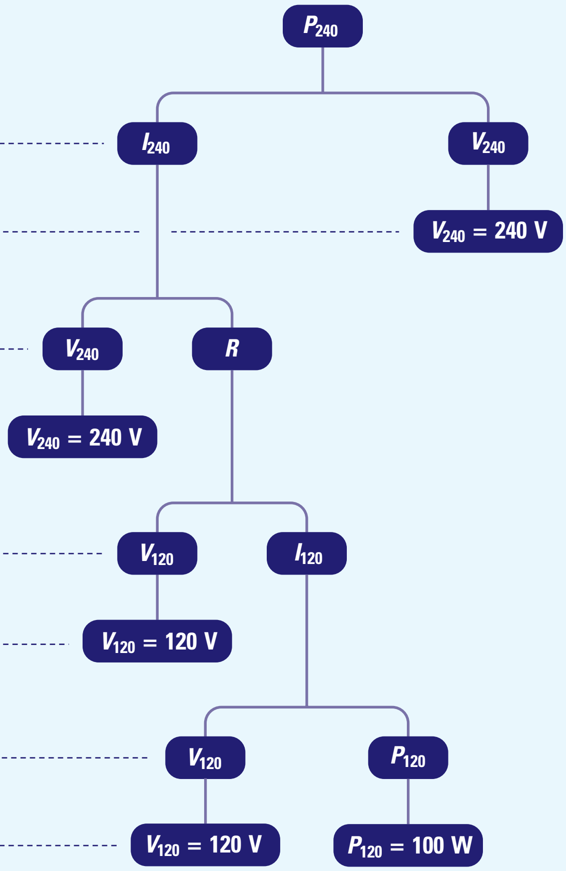
The potential difference is known.

You can find the resistance if you know the current and potential difference at 120 V.

The potential difference is known.

You can find the current at 120 V if you know the potential difference and power at 120 V.

The potential difference and the power at 120 V are known.



The problem is solved. Now do the calculations.

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Strategy	Calculations
Find the current at 120 V.	$P_{120} = I_{120}V_{120}$
	$I_{120} = \frac{P_{120}}{V_{120}}$
	$I_{120} = \frac{100\ \text{W}}{120\ \text{V}}$
	$I_{120} = 0.833\ \text{A}$
Find the resistance.	$R = \frac{V_{120}}{I_{120}}$
	$R = \frac{120\ \text{V}}{0.833\ \text{A}}$
	$R = 144\ \Omega$
Find the current at 240 V.	$I_{240} = \frac{V_{240}}{R}$
	$I_{240} = \frac{240\ \text{V}}{144\ \Omega}$
	$I_{240} = 1.67\ \text{A}$
Find the power output at 240 V.	$P_{240} = I_{240}V_{240}$
	$P_{240} = (1.666\ \text{A})(240\ \text{V})$
	$P_{240} = 400\ \text{W}$

The power output at 240 V is 400 W. Notice that the power output is four times as great when you double the potential difference across a given resistance. Such an increase in power output would overheat the filament and melt it.

Validate

Although it might seem strange that the power output quadruples when the potential difference doubles, it is logical. Doubling the potential difference for a given resistance doubles the current. When both potential difference and current are doubled, the power output is quadrupled.

PRACTICE PROBLEMS

43. **(a)** What is the power output of a 45 Ω resistance when connected to
- a 180 V power supply?
- a 270 V power supply?

(b) Find the ratio of the potential differences and the ratio of the power outputs. What is the relationship between the two ratios?

44. A load has a power rating of 160 W when the current in it is 6.0 A. What will be the power output if the current increases to 15 A?

45. **(a)** What is the power output of a circuit that consists of a 25 Ω resistance when connected to a 100 V supply?

(b) If a second 25 Ω resistance is connected in series with the first, what will be the power output of the circuit? Why has the power output of the circuit decreased?

You probably noticed in the model problem and practice problems on the preceding pages that the resistance of a device is often the key to finding the power output. It would be convenient if relationships were available that would relate power output, resistance, and potential difference (P, R, and V) or power output, resistance, and current (P, R, and I). Since Ohm's law relates resistance, current, and potential difference, you can use it to develop more relationships for power.

- Start with the relationship between potential difference, current, and power. $P = IV$
- To eliminate current and to introduce resistance, write Ohm's law and solve for current. $V = IR$ $$I = \frac{V}{R}$$
- Substitute $\frac{V}{R}$ in place of I in the equation for power. $$P = \frac{V}{R}V$$
- Simplify. $$P = \frac{V^2}{R}$$

To find a relationship between P, R, and I, use a similar procedure, but eliminate V from the equation for power.

- Start with the first equation for power. $P = IV$
- Express Ohm's law in terms of potential difference. $V = IR$
- Substitute IR in place of V in the equation for power. $P = I(IR)$
- Simplify. $P = I^2R$

ALTERNATIVE EQUATIONS FOR POWER

Power is the quotient of the square of the potential difference and the resistance.

$$P = \frac{V^2}{R}$$

Power is the product of the square of the current and the resistance.

$$P = I^2R$$

Quantity	Symbol	SI unit
power	P	W (watt)
potential difference	V	V (volt)
resistance	R	Ω (ohm)
current	I	A (ampere)

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Unit Analysis

Recall from the definitions of current and potential difference

$$\mathrm{A} = \frac{\mathrm{C}}{\mathrm{s}} \qquad \mathrm{V} = \frac{\mathrm{J}}{\mathrm{C}}$$

From Ohm's law, $V = \mathrm{A} \cdot \Omega$, therefore $\frac{\mathrm{V}}{\Omega} = \mathrm{A}$.

Use these relationships to analyze the units for the two power formulas.

$$(\text{watt}) = \frac{(\text{volt})^2}{(\text{ohm})}$$
$$\mathrm{W} = \frac{\mathrm{V}^2}{\Omega}$$
$$\mathrm{W} = \left(\frac{\mathrm{V}}{\Omega}\right) V$$
$$\mathrm{W} = \mathrm{A} \cdot \mathrm{V}$$
$$\mathrm{W} = \left(\frac{\cancel{\mathrm{C}}}{\mathrm{s}}\right)\left(\frac{\mathrm{J}}{\cancel{\mathrm{C}}}\right)$$
$$\mathrm{W} = \mathrm{W}$$

$$(\text{watt}) = (\text{ampere})^2(\text{ohm})$$
$$\mathrm{W} = (\mathrm{A})^2(\Omega)$$
$$\mathrm{W} = (\mathrm{A})(\mathrm{A} \cdot \Omega)$$
$$\mathrm{W} = (\mathrm{A})(\mathrm{V})$$
$$\mathrm{W} = \left(\frac{\cancel{\mathrm{C}}}{\mathrm{s}}\right)\left(\frac{\mathrm{J}}{\cancel{\mathrm{C}}}\right)$$
$$\mathrm{W} = \frac{\mathrm{J}}{\mathrm{s}}$$
$$\mathrm{W} = \mathrm{W}$$

MODEL PROBLEM

Resistance and Power

An electric kettle is rated at 1500 W for a 120 V potential difference.

(a) What is the resistance of the heating element of the kettle?

(b) What will be the power output if the potential difference falls to 108 V?

Frame the Problem

- An electric kettle will use *energy* at a *rate* of 1500 W if connected to a 120 V line.
- *Power*, *potential difference*, *current*, and *resistance* are all related.

Identify the Goal

The resistance, R, of the heating element of the kettle
The power, P, if the potential difference decreases

Variables and Constants

Known

$P_1 = 1500$ W

$V_1 = 120$ V

$V_2 = 108$ V

Unknown

R

P_2

Strategy

Use the relationship among power, potential difference, and resistance.

Calculations

$$P_1 = \frac{V_1^2}{R}$$

Substitute first

$$1.50 \times 10^3 \text{ W} = \frac{(1.20 \times 10^2 \text{ V})^2}{R}$$

$$(1.50 \times 10^3 \text{ W})(R) = \frac{(1.20 \times 10^2 \text{ V})^2}{\cancel{R}}(\cancel{R})$$

$$\frac{\cancel{(1.50 \times 10^3 \text{ W})}(R)}{\cancel{1.50 \times 10^3 \text{ W}}} = \frac{(1.20 \times 10^2 \text{ V})^2}{1.50 \times 10^3 \text{ W}}$$

$$R = 9.60 \frac{\text{V}^2}{\text{W}}$$

$$R = 9.60\ \Omega$$

Solve for *R* first

$$P_1 R = \frac{V_1^2}{\cancel{R}}\cancel{R}$$

$$P_1 R = V_1^2$$

$$\frac{\cancel{P_1} R}{\cancel{P_1}} = \frac{V_1^2}{P_1}$$

$$R = \frac{V_1^2}{P_1}$$

$$R = \frac{(1.2 \times 10^2 \text{ V})^2}{1.50 \times 10^3 \text{ W}}$$

$$R = 9.60 \frac{\text{V}^2}{\text{W}}$$

$$R = 9.60\ \Omega$$

Unit Check: $\frac{\text{V}^2}{\text{W}} = \frac{\frac{\text{J}^2}{\text{C}^2}}{\frac{\text{J}}{\text{s}}} = \left(\frac{\text{J} \cdot \cancel{\text{J}}}{\text{C}^2}\right)\left(\frac{\text{s}}{\cancel{\text{J}}}\right) = \left(\frac{\text{J}}{\text{C}}\right)\left(\frac{\text{s}}{\text{C}}\right) = V\frac{1}{\text{A}} = \Omega$

(a) The resistance of the coil in the kettle is 9.60 Ω.

Find the power output when the potential difference is 108 V.

$$P_2 = \frac{V_2^2}{R}$$

$$P_2 = \frac{(108 \text{ V})^2}{9.60\ \Omega}$$

$$P_2 = 1215 \frac{\text{V}^2}{\Omega}$$

$$P_2 = 1.22 \times 10^3 \text{ W}$$

See the box on the previous page for a unit analysis.

(b) The power output drops to 1.22×10^3 W when the potential difference drops to 108 V.

(**Note:** This is a 10% loss of potential difference and a 19% loss in power. The percentage decrease in power is nearly double that of the decrease in potential difference because power is proportional to the square of the potential difference. Household voltage is rated at 120 V but it can fluctuate more than 5%.)

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Validate

The units combine properly to give ohms in part (a), and watts in part (b). From past experience, you would expect a proportionately larger drop in the power output than in potential difference.

PRACTICE PROBLEMS

46. A filament in a light bulb rated at 192 W, has a resistance of 12.0 Ω. Calculate the potential difference at which the bulb is designed to operate.

47. An electric kettle is rated at 960 W when operating at 120 V. What must be the resistance of the heating element in the kettle?

48. If a current of 3.50 A is flowing through a resistance of 24.0 Ω, what is the power output?

49. A toaster that has a power rating of 900 W (9.00×10^2 W) draws a current of 7.50 A. If 2.40×10^5 J of electric energy are consumed while toasting some bread, calculate how much charge passed through the toaster.

50. A floodlight filament has an operating resistance of 22.0 Ω. The lamp is designed to operate at 110 V.

(a) What is its power rating?

(b) How much energy is consumed if you use the lamp for 2.50 hours?

Energy Consumption

The electric meter at your home gives readings in **kilowatt-hours** (kW · h). One kilowatt-hour represents the energy transformed by a power output of 1000 W for one hour. This is equivalent to 3.6×10^6 J (3.6 MJ). A typical charge by an energy company for energy consumed might be $0.07 per kW · h. That means that for only seven cents, you can buy enough energy to lift 360 kg a vertical distance of more than 1 km.

Figure 15.28 The electric meter in your home measures how much energy (in kW · h) that you use.

MISCONCEPTION

Power or Energy?

Often you hear people speaking of buying power from the "power" company. If you check your monthly electric bill, you will see that you are charged for the energy (measured in kilowatt-hours) that you consumed. Power measures how fast you do the work, not how much work you do. It is electric energy that keeps your lights on and runs your television. People should say that they buy energy from the "energy" company.

$$1\ \text{kW} \cdot \text{h} = (1000\ \text{W})(1\ \text{h})$$
$$\text{kW} \cdot \text{h} = (1 \times 10^3\ \text{W})(3.6 \times 10^3\ \text{s})$$
$$\text{kW} \cdot \text{h} = 3.6 \times 10^6\ \text{W} \cdot \text{s}$$
$$\text{kW} \cdot \text{h} = 3.6 \times 10^6\ \text{J}$$

(or 3.6 MJ of energy)

MODEL PROBLEM

The Cost of Watching Television

A family has its television set on for an average of 4.0 h per day. If the television set is rated at 80 W and energy costs $0.070 per kW · h, how much would it cost to operate the set for 30 days?

Frame the Problem

- The total amount of *electric energy* a television set uses in a specific amount of *time* depends on its *power output*.
- Power companies charge a specific amount of *dollars* per unit of *energy* used.

Identify the Goal

The cost, in dollars, for operating a television set for 30 days

Variables and Constants

Known

$\Delta t = (4.0\frac{\text{h}}{\text{day}})(30\ \text{days})$

$\text{rate} = \frac{\$0.070}{\text{kW} \cdot \text{h}}$

$P = 80\ \text{W}$

Unknown

Cost

ΔE_{Q}

Strategy

Find the total time, in hours, that the television set typically is on during one 30-day period.

Find the total amount of energy consumed by using the definition for power.

Find the cost.

The cost of operating a television set for 4.0 h per day is $0.67 for 30 days.

Calculations

$$\Delta t = \left(4.0\frac{\text{h}}{\cancel{\text{day}}}\right)(30\ \cancel{\text{days}})$$

$$\Delta t = 120\ \text{h}$$

$$P = \frac{\Delta E_{\text{Q}}}{\Delta t}$$

Substitute first

$$(80\ \text{W})(120\ \text{h}) = \frac{\Delta E_{\text{e}}}{\cancel{120\ \text{h}}}\cancel{120\ \text{h}}$$

$$\Delta E_{\text{Q}} = 9600\ \text{W} \cdot \text{h}$$

Solve for ΔE_{Q} first

$$(P)(\Delta t) = \frac{\Delta E_{\text{Q}}}{\cancel{\Delta t}} = \cancel{\Delta t}$$

$$(P)(\Delta t) = \Delta E_{\text{Q}}$$

$$\Delta E_{\text{Q}} = (80\ \text{W})(120\ \text{h})$$

$$\Delta E_{\text{Q}} = 9600\ \text{W} \cdot \text{h}$$

$$\text{Cost} = \text{rate} \cdot \Delta E_{\text{Q}}$$

$$Cost = \frac{\$0.070}{\cancel{\text{kW}} \cdot \cancel{\text{h}}}\ 9600\ \cancel{\text{W}} \cdot \cancel{\text{h}}\ \frac{1\ \cancel{\text{kW}}}{1000\ \cancel{\text{W}}}$$

$$Cost = \$0.67$$

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Validate

The units combine and cancel to give dollars. At first, the cost seems to be low. However, a television set does not use as much energy as many other appliances. As well, electric energy is relatively inexpensive.

PRACTICE PROBLEMS

51. It takes 25.0 min for your clothes dryer to dry a load of clothes. If the energy company charges 7.20 cents per $kW \cdot h$, how much does it cost to dry a load of clothes in a 1250 W dryer?

52. It takes 12.0 min to dry your hair using a blow dryer that has a resistance of 21.0 Ω and draws a current of 5.50 A. Calculate the cost to dry your hair if electricity costs 8.50 cents per $kW \cdot h$.

53. An electric kettle, which operates on a 120 V supply, has a heating element with a resistance of 10.0 Ω. If it takes 3.2 min to boil a litre of water and the cost of energy is 6.5 cents per $kW \cdot h$, calculate

(a) the power rating of the kettle

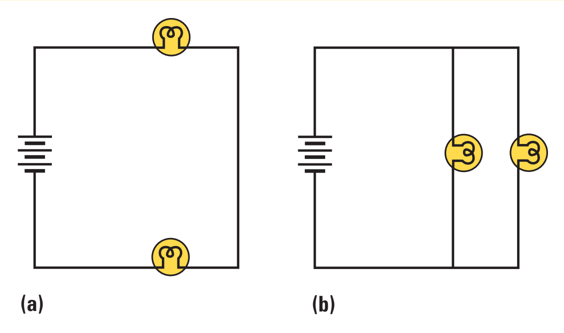
(b) the cost to boil the water

15.5 Section Review

1. K/U Write four different equations that could be used to find the power consumed by a circuit element. Explain why it is useful to have so many different equations for power.

2. C The brightness of a light bulb is directly related to the power it consumes. In the circuits in the diagram on the right, the light bulbs have the same ratings. In which circuit will the light bulbs glow most brightly? Explain your reasoning.

3. I Using unit analysis, show that a kilowatt-hour is a unit of energy.



UNIT PROJECT PREP

Think about these questions in the analysis of your motor.

- How powerful will your electric motor be?
- What factors will affect the power output of your motor?
- How can you determine the power of your motor?