Pre-Calculus 120 A


## Polar Coordinates

In a Cartesian coordinate system we identify each point in the plane according to a unique ordered pair of real numbers ( $x, y$ ).

We can also locate a point according to its distance from the origin (pole) and rotation from the positive $x$-axis (polar axis). As illustrated in the diagram to the right, if $P(x, y)$ is a point in the plane $r$ units from the pole and a rotation $\theta$ from the polar axis, then the ordered pair ( $x, y$ ) can also be identified by $(\mathrm{r}, \boldsymbol{\theta})$. These coordinates are called the polar coordinates of P .


CONVERTING FROM POLAR COORDINATES $(r, \theta)$ TO RECTANGULAR COORDINATES $(x, y)$ :

- Find x using $\mathrm{x}=\mathrm{r} \cos \theta$ (Since $\cos \theta=\frac{x}{r}$ )
- Find y using $\mathrm{y}=\mathrm{r} \sin \theta \quad\left(\right.$ Since $\sin \theta=\frac{y}{r}$ )
- The rectangular coordinates are $(x, y)$


## Example 1: Converting from Polar Coordinates to Rectangular Coordinates

Sketch a diagram and convert the following polar coordinates to exact rectangular coordinates.
a. $\left(6,135^{\circ}\right)$
b. $\left(-5, \frac{5 \pi}{3}\right)$
c. $\left(7,-150^{\circ}\right)$

## Solution:



CONVERTING FROM RECTANGULAR COORDINATES $(x, y)$ TO POLAR COORDINATES $(r, \theta)$ :

- Find the radius, $r$, using the Pythagorean relationship $r=\sqrt{x^{2}+y^{2}}$.
- Find the reference angle, $\theta_{R}$, using $\tan \theta_{R}=\left|\frac{y}{x}\right|$.
- Find the angle, $\theta$, by determining the quadrant in which the terminal arm is located and using the reference angle, $\theta_{R}$.
- The polar coordinates are $(r, \theta)$.


## Example 2: Converting from Rectangular Coordinates to Polar Coordinates

Sketch a diagram and convert the following rectangular coordinates to polar coordinates.
a. $P(8,15)$
b. $Q(7,-24)$

Solution:

| a. $\mathrm{P}(8,15)$ | b. $\mathrm{Q}(7$, -24) |
| :--- | :--- |
|  |  |
| $(\mathrm{r}, \theta)=\ldots$ | $(\mathrm{r}, \theta)=$ |

