Transformations of Logarithmic Functions

- The graph of the logarithmic function $y = a\log_c(b(x-h))+k$ can be obtained by transforming the graph of $y = \log_c x$. These transformations should be performed in the same manner as those applied to any other function.

Example 1: Translations of a Logarithmic Function

Sketch the graph of $y = \log_4(x+4)-5$ and state the mapping rule, domain and range, x- and y- intercepts, and equation of the asymptote.

Solution:

Begin with the graph of $y = \log_4 x$. Think of $y = \log_4 x$ as $4^y = x$. Choose "nice" values of y first and then determine the x-values. Next, identify the transformations on this function to create $y = \log_4(x+4)-5$.

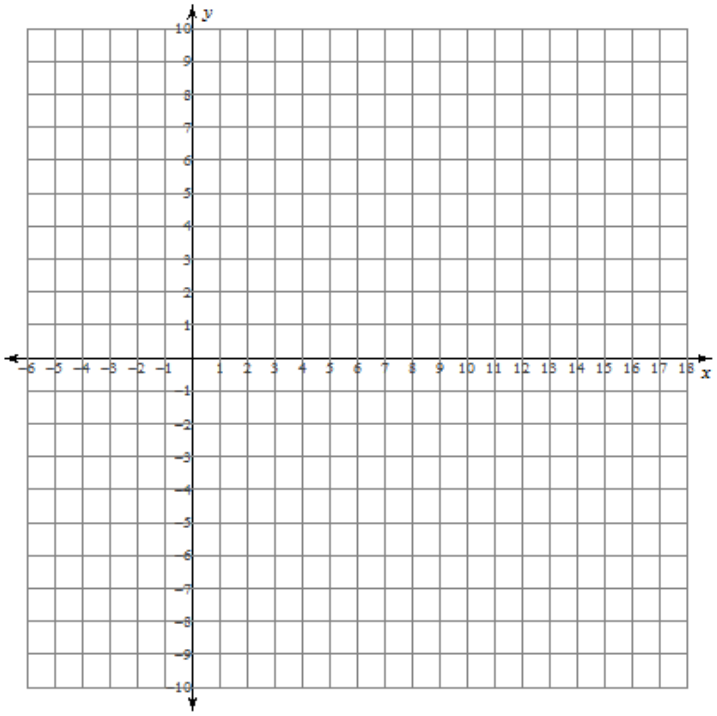
- The base graph must be translated _____.

  Mapping rule: $(x,y) \rightarrow$ _____.

- Complete each table of values and sketch the graphs of both functions.

$y = \log_4 x$	
x	y

$y = \log_4(x+4)-5$	
x	y



For the function $y = \log_4(x+4)-5$:

Domain: _____

Range: _____

x-intercept: _____

y-intercept: _____

Equation of the vertical asymptote: _____

Example 2: Reflections and Stretches of Logarithmic Functions

Sketch the graph of $y = -\log_2 4x$ and state the mapping rule, domain and range, x- and y- intercepts, and equation of the asymptote.

Solution:

Begin with the graph of $y = \log_2 x$. Think of $y = \log_2 x$ as $2^y = x$. Choose "nice" values of y first and then determine the x-values. Next, identify the transformations on this function to create $y = -\log_2 4x$.

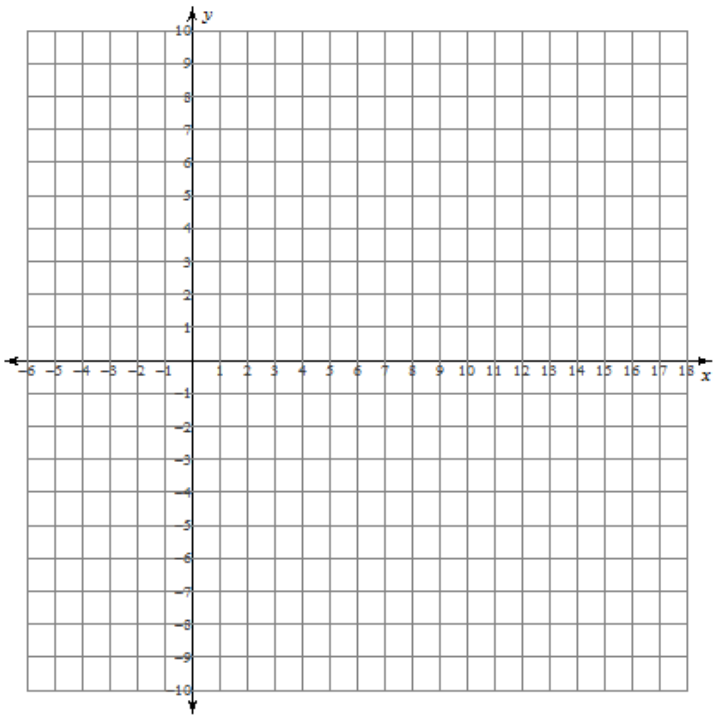
- The base graph must be _____.

  Mapping rule: $(x, y) \rightarrow$ _____.

- Complete each table of values and sketch the graphs of both functions.

$y = \log_2 x$	
x	y

$y = -\log_2 4x$	
x	y



For the function $y = -\log_2 4x$:

Domain: _____

Range: _____

x-intercept: _____

y-intercept: _____

Equation of the vertical asymptote: _____

Example 3: Combine Transformations

Sketch the graph of $y = -2\log_3(x-3)+5$ and state the mapping rule, domain and range, x- and y- intercepts, and equation of the asymptote.

Solution:

Begin with the graph of $y = \log_3 x$. Think of $y = \log_3 x$ as $3^y = x$. Choose "nice" values of y first and then determine the x-values. Next, identify the transformations on this function to create $y = -2\log_3(x-3)+5$.

- The base graph must be _____

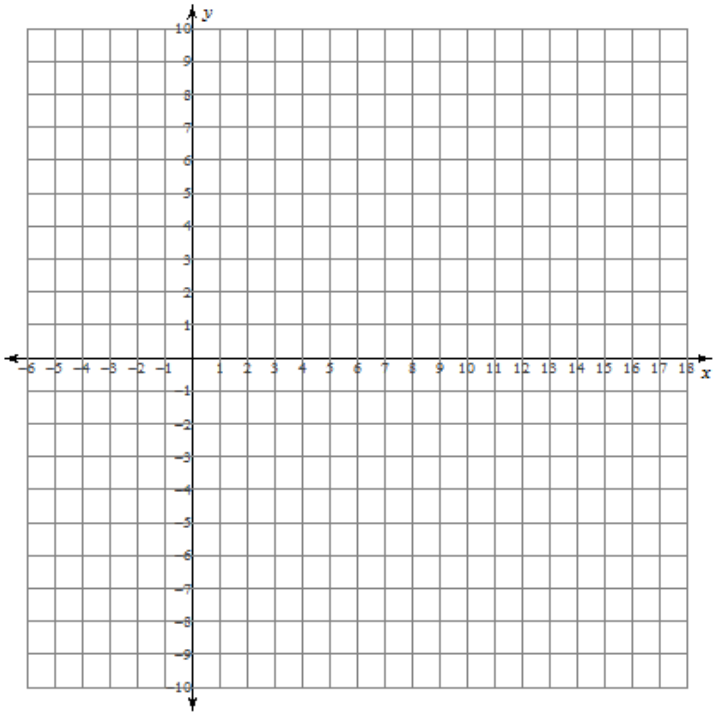
  _____.

  Mapping rule: $(x, y) \rightarrow$ _____.

- Complete each table of values and sketch the graphs of both functions.

$y = \log_3 x$	
x	y

$y = -2\log_3(x-3)+5$	
x	y



For the function $y = -2\log_3(x-3)+5$:

Domain: _____

Range: _____

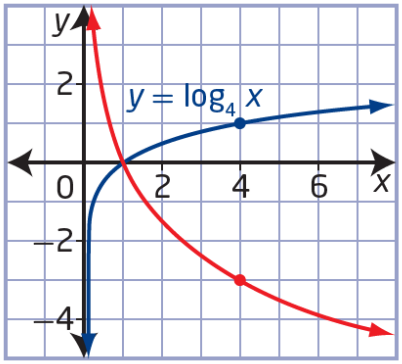
x-intercept: _____

y-intercept: _____

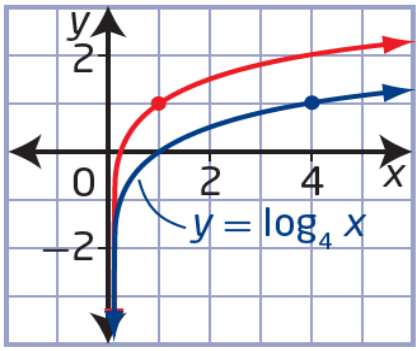
Equation of the vertical asymptote: _____

Example 4: Determine the Equation of a Logarithmic Function Given Its Graph

a. The transformed graph illustrated in the diagram below can be generated by stretching and reflecting the graph of $y = \log_4 x$. Determine the equation of the transformed graph.



b. The transformed graph illustrated in the diagram below can be generated by stretching the graph of $y = \log_4 x$. Determine the equation of the transformed graph.



Solution:

a. _____

b. _____

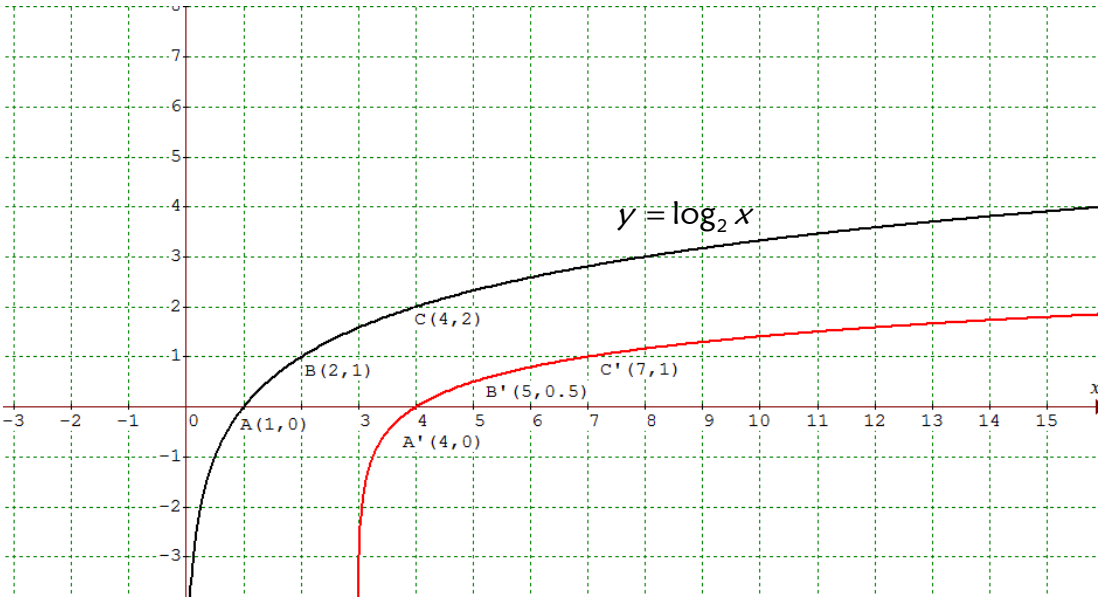
Example 5: Use Transformations of an Exponential Function to Model a Situation

There is a logarithmic relationship between butterflies and flowers. In one study, scientists found that the relationship between the number, F, of flower species that a butterfly feeds on and the number, B, of butterflies observed can be modeled by the function $F = -2.641 + 8.958 \log B$.

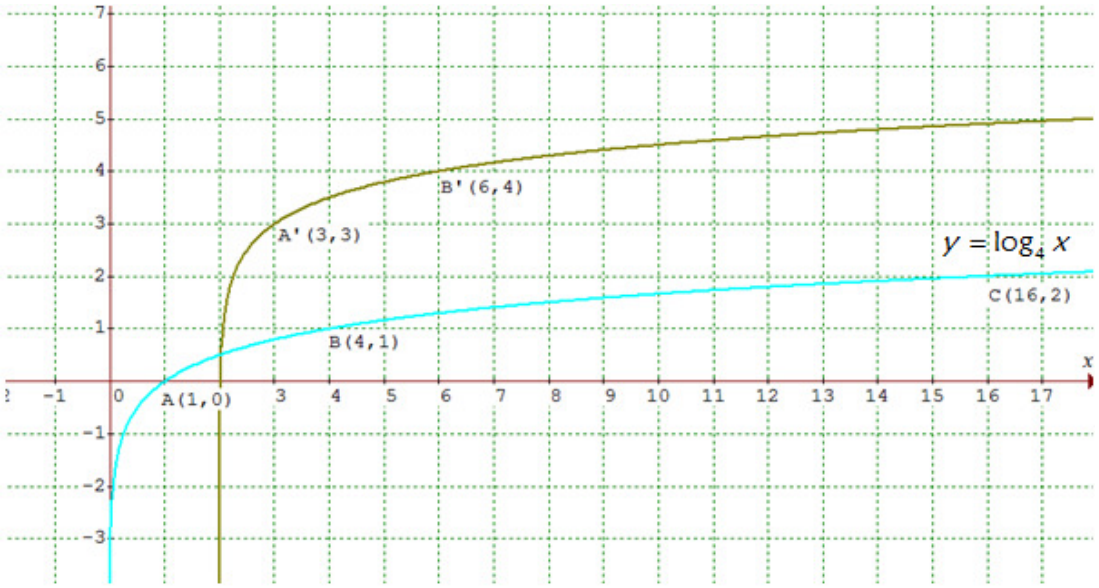
Predict the number of butterfly observations in a region with 25 flower species.

EXTRA PRACTICE:

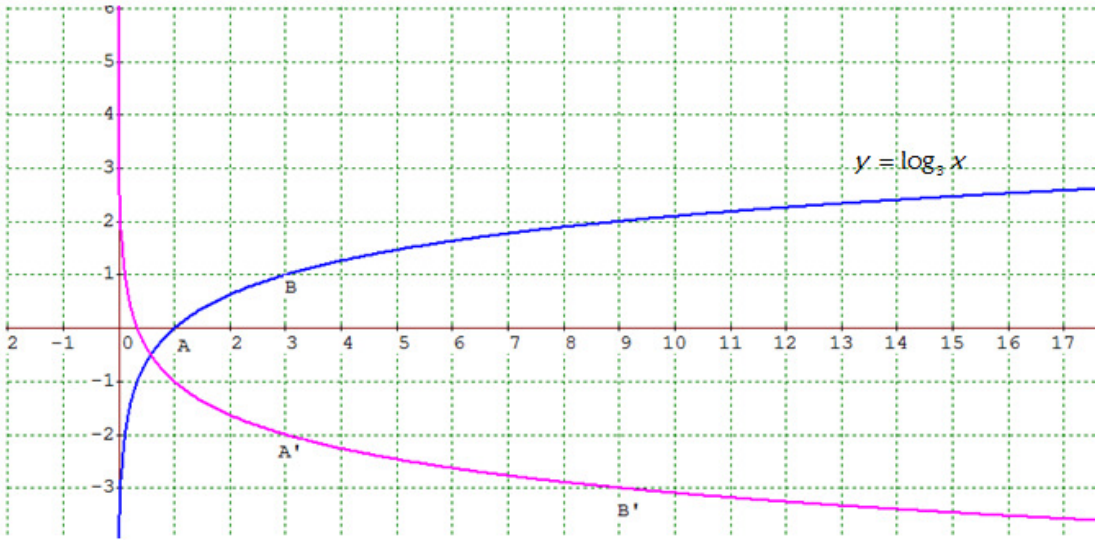
The transformed graph shown on the grid can be generated by stretching and translating the graph of $y = \log_2 x$. Determine the equation of the transformed graph.



The transformed graph shown on the grid can be generated by translating the graph of $y = \log_4 x$. Determine the equation of the transformed graph.



The transformed graph shown on the grid can be generated by reflecting, stretching and translating the graph of $y = \log_3 x$. Determine the equation of the transformed graph.



The transformed graph shown on the grid can be generated by reflecting, stretching and translating the graph of $y = \log_5 x$. Determine the equation of the transformed graph.