Reflections and Stretches

Reflection: *a transformation where each point of the original graph has an image point resulting from a reflection in a line. A reflection may result in a change of orientation of a graph while preserving its shape.*

Stretch: *a transformation in which the distance of each x-coordinate or y-coordinate from the line of reflection is multiplied by some scale factor. Scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection. A stretch changes the shape of the graph but not its orientation.*

Invariant Point: *a point on a graph that remains unchanged after a transformation is applied to it. Any point on a line of reflection is an invariant point.*

Example 1: Compare the Graphs of y = f(x), y = –f(x), and y = f(– x)

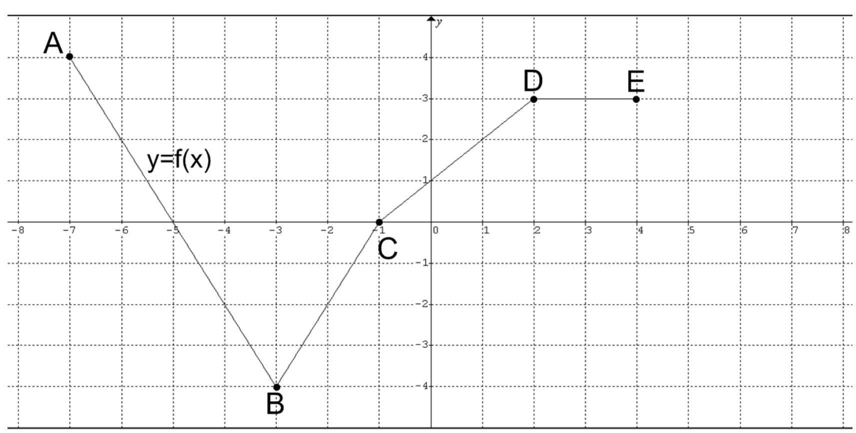
a. Given the graph of $y = f(x)$, graph the function $y = -f(x)$. How is the graph of $y = -f(x)$ related to the graph of $y = f(x)$?

Solution:

Use key points on the graph of $y = f(x)$ to create a table of values.

The image points on the graph of $y = -f(x)$ have the same x-coordinates but different y-coordinates. Multiply the y-coordinates of points on the graph of $y = f(x)$ by –1.

	x	$y = f(x)$
A	–7	4
B	–3	–4
C	–1	0
D	2	3
E	4	3
	x	$y = -f(x)$
A’	–7	
B’	–3	
C’	–1	
D’	2	
E’	4	



The graph of $y = -f(x)$ is congruent to the graph of $y = f(x)$.

The points on the graph of $y = f(x)$ relate to the points on the graph of $y = -f(x)$ by the mapping ______________________________.

The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ in the ___________________.

The points _______________ and _______________ are *invariant* points.

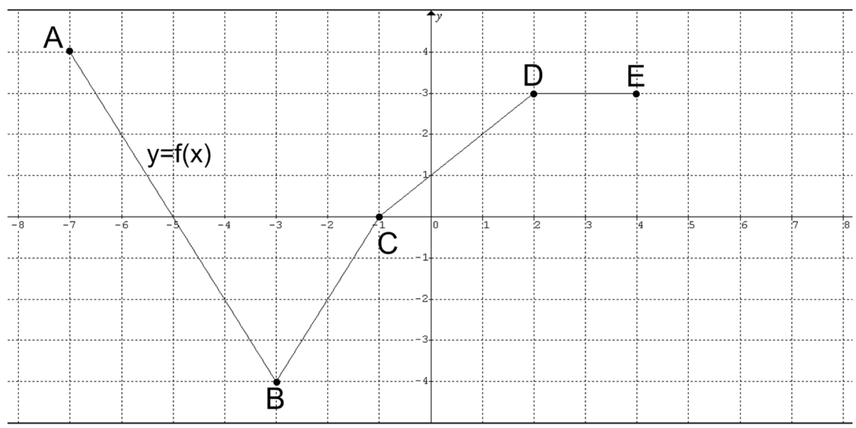
b. Given the graph of $y = f(x)$, graph the function $y = f(-x)$. How is the graph of $y = f(-x)$ related to the graph of $y = f(x)$?

Solution:

Use key points on the graph of $y = f(x)$ to create a table of values.

The image points on the graph of $y = f(-x)$ have the same y-coordinates but different x-coordinates. Multiply the x-coordinates of points on the graph of $y = f(x)$ by –1.

	x	$y = f(x)$
A	–7	4
B	–3	–4
C	–1	0
D	2	3
E	4	3
	x	$y = f(-x)$
A'		4
B'		–4
C'		0
D'		3
E'		3



The graph of $y = f(-x)$ is congruent to the graph of $y = f(x)$.

The points on the graph of $y = f(x)$ relate to the points on the graph of $y = f(-x)$ by the mapping ____________________________.

The graph of $y = f(-x)$ is a reflection of the graph of $y = f(x)$ in the _________________.

The point _____________ is an invariant point.

Example 2: Vertical Stretches

Given the graph of $y = f(x)$,

- transform the graph of f(x) to sketch the graph of g(x)
- describe the transformation
- state any invariant points
- state the domain and range of the functions

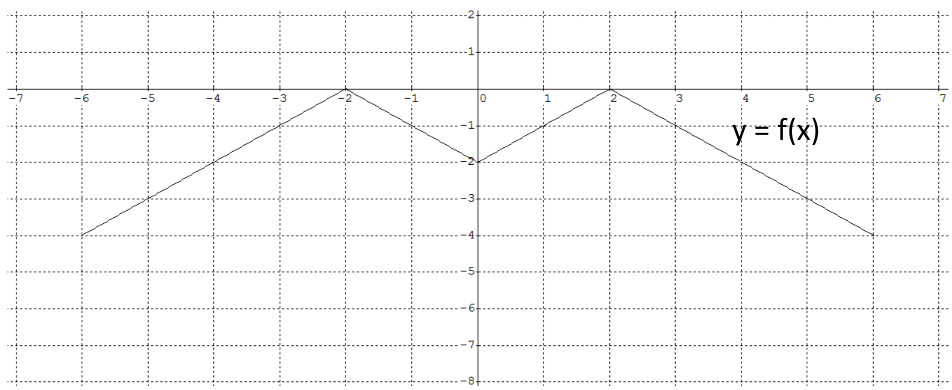
a. $g(x) = 2f(x)$

b. $g(x) = \frac{1}{2}f(x)$

Solution:

a. Use the key points on the graph of $y = f(x)$ to create a table of values. The image points on the graph of $g(x) = 2f(x)$ have the same x-coordinates but different y-coordinates. Multiply the y-coordinates of points on the graph of $y = f(x)$ by 2.

x	$y = f(x)$	$y = g(x) = 2f(x)$
–6		
–2		
0		
2		
6		



The points on the graph of $f(x)$ relate to the points on the graph of $g(x) = 2f(x)$ by the mapping ______________________________.

The graph of $g(x) = 2f(x)$ is a vertical stretch of the graph of $f(x)$ about the ________________ by a factor of 2.

The invariant points are ______________ and ______________.

The domain of f(x) is _________________________________ or ________________

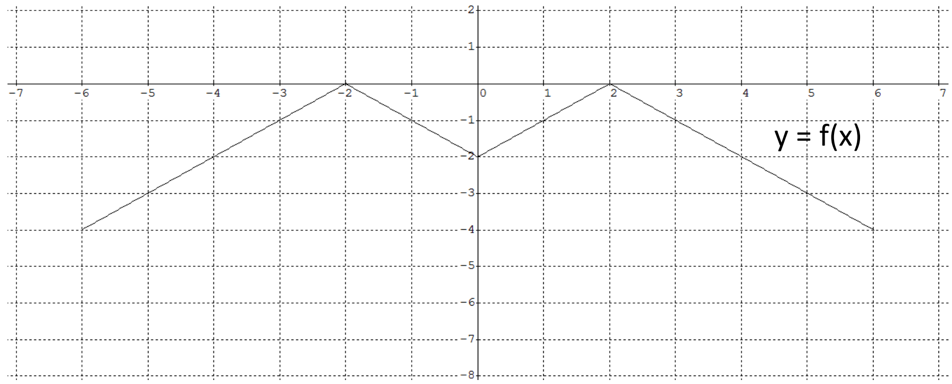
and the range is _________________________________ or ________________.

The domain of g(x) is _________________________________ or ________________

and the range is _________________________________ or ________________.

b. Use the key points on the graph of $y = f(x)$ to create a table of values. The image points on the graph of $g(x) = \frac{1}{2}f(x)$ have the same x-coordinates but different y-coordinates. Multiply the y-coordinates of points on the graph of $y = f(x)$ by ½.

x	$y = f(x)$	$y = g(x) = \frac{1}{2}f(x)$
–6	–4	
–2	0	
0	–2	
2	0	
6	–4	



The points on the graph of $f(x)$ relate to the points on the graph of $g(x) = \frac{1}{2}f(x)$ by the mapping

____________________________.

The graph of $g(x) = \frac{1}{2}f(x)$ is a vertical stretch of the graph of $f(x)$ about the _____________ by a factor of ½.

The invariant points are _____________ and _____________.

The domain of f(x) is ______________________________ or _______________

and the range is ______________________________ or _______________.

The domain of g(x) is ______________________________ or _______________

and the range is ______________________________ or _______________.

Example 3: Horizontal Stretches

Given the graph of $y = f(x)$,

- transform the graph of f(x) to sketch the graph of g(x)
- describe the transformation
- state any invariant points
- state the domain and range of the functions

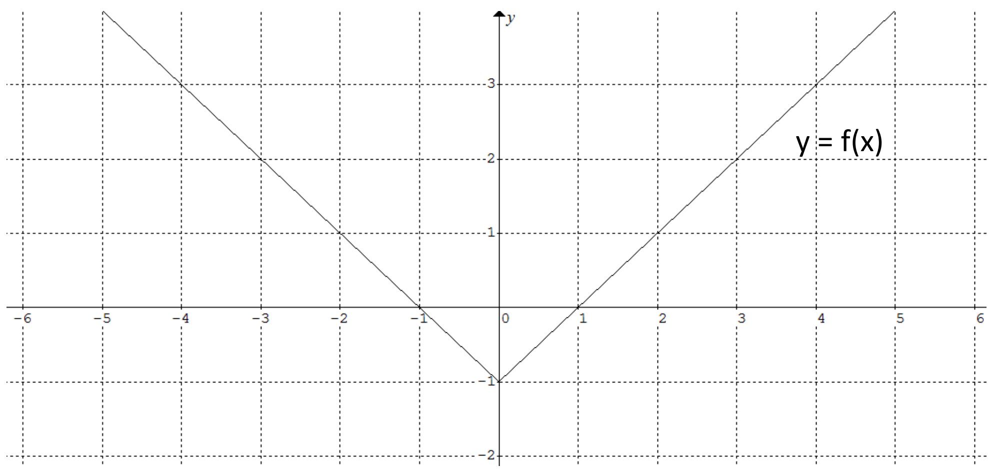
a. $g(x) = f(2x)$

b. $g(x) = f(\frac{1}{2}x)$

Solution:

a. Use key points on the graph of $y = f(x)$ to create a table of values. The image points on the graph of $g(x) = f(2x)$ have the same y-coordinates but different x-coordinates. Multiply the x-coordinates of points on the graph of $y = f(x)$ by ½.

x	$y = f(x)$
	3
	1
	–1
	1
	3
x	$y = g(x) = f(2x)$
	3
	1
	–1
	1
	3



The points on the graph of $f(x)$ relate to the points on the graph of $g(x) = f(2x)$ by the mapping ____________________________.

The graph of $g(x) = f(2x)$ is a horizontal stretch of the graph of $f(x)$ about the ___________ by a factor of ½.

The invariant point is _____________.

The domain of f(x) is ______________________________ or _______________

and the range is ______________________________ or _______________.

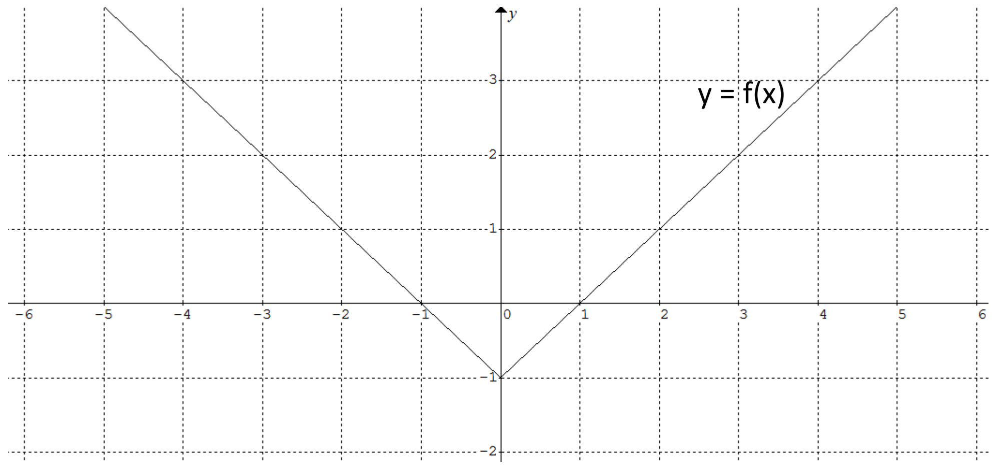
The domain of g(x) is ______________________________ or _______________

and the range is ______________________________ or _______________.

b. Use key points on the graph of $y = f(x)$ to create a table of values. The image points on the graph of $g(x) = f(\frac{1}{2}x)$ have the same y-coordinates but different x-coordinates. Multiply the x-coordinates of points on the graph of $y = f(x)$ by 2.

x	$y = f(x)$
	2
	1
	0
	–1
	0
	1
	2

x	$y = g(x) = f(\frac{1}{2}x)$
	2
	1
	0
	–1
	0
	1
	2



The points on the graph of $f(x)$ relate to the points on the graph of $g(x) = f(\frac{1}{2}x)$ by the mapping

______________________________.

The graph of $g(x) = f(\frac{1}{2}x)$ is a horizontal stretch of the graph of $f(x)$ about the ____________ by a factor of 2.

The invariant point is ______________.

The domain of f(x) is _________________________________ or ________________

and the range is _________________________________ or ________________.

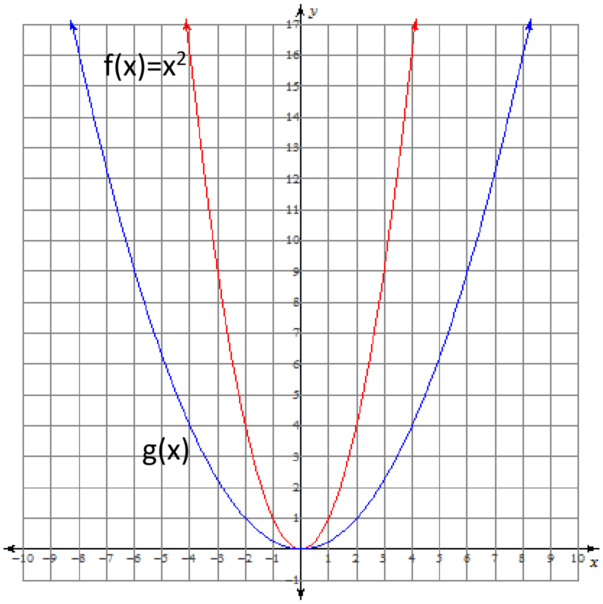
The domain of g(x) is _________________________________ or ________________

and the range is _________________________________ or ________________.

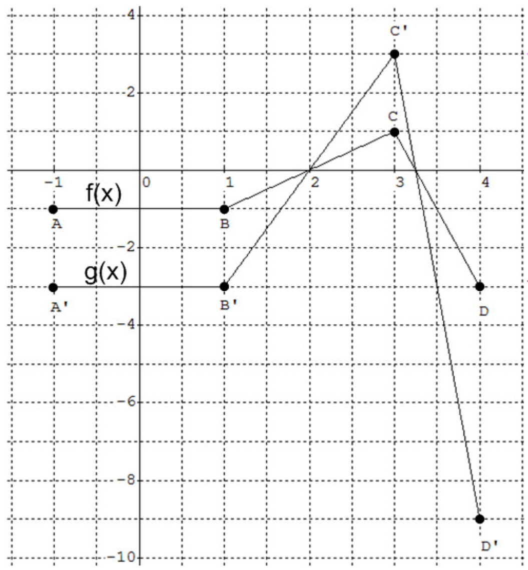
Example 4: Write the Equation of a Transformed Function

The graph of the function y=f(x) has been transformed by either a stretch or a reflection. Write the equation of the transformed graph, g(x).

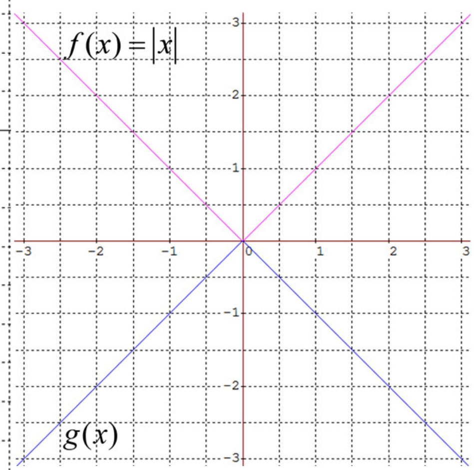
a.



b.



c.



Solution:

a. The shape has changed so the graph has been transformed by a stretch. In this case, the stretch can be described in two ways.

Case 1: Check for a pattern in the y-coordinates.

x	y = f(x)	y = g(x)
–4		
–2		
0		
2		
4		

A vertical stretch results when the vertical distances of the transformed graph are a constant multiple of those of the original graph with respect to the x-axis.

The transformation can be described by the mapping ____________________.

The equation of the transformed function is g(x) = __________ or g(x) = __________.

Case 2: Check for a pattern in the x-coordinates.

x	y = f(x)		x	y = g(x)
	16			16
	4			4
	1			1
	0			0
	1			1
	4			4
	16			16

A horizontal stretch results when the horizontal distances of the transformed graph are a constant multiple of those of the original graph with respect to the y-axis.

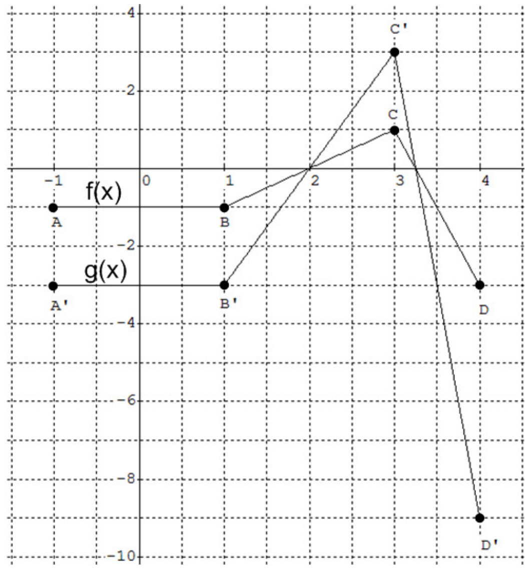
The transformation can be described by the mapping ____________________.

The equation of the transformed function is g(x) = __________ or g(x) = __________.

b. The shape has changed so the graph has been transformed by a stretch. In this case, the stretch can be described as a vertical stretch.

Check for a pattern in the y-coordinates.

x	y = f(x)	y = g(x)

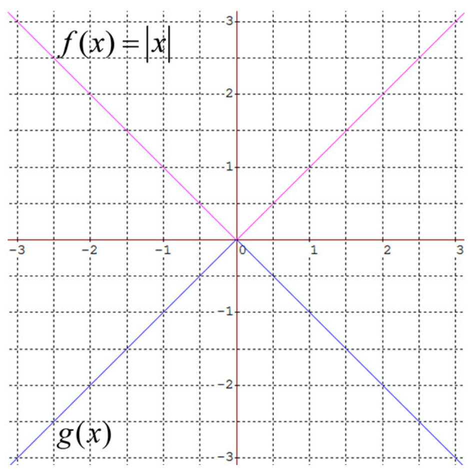


The transformation can be described by the mapping _________________________.

The equation of the transformed function is g(x) = _______________.

c. The shape of the graph has not changed. The graph has been transformed by a reflection in the ___________.

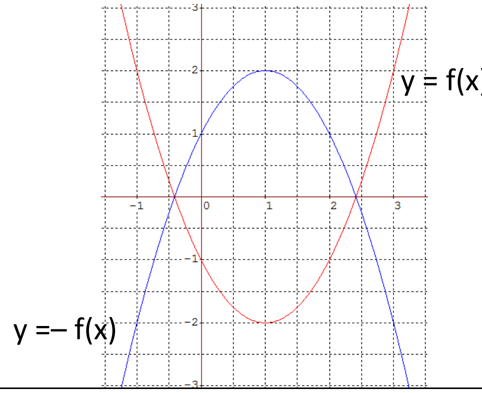
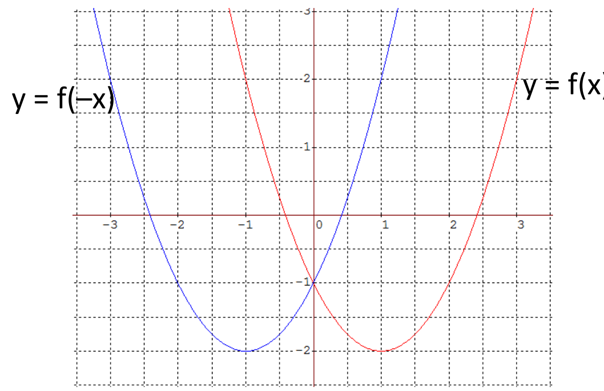
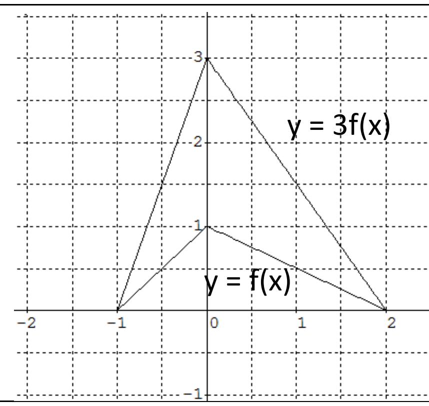
x	y = f(x)	y = g(x)



The transformation can be described by the mapping ____________________.

The equation of the transformed function is g(x) = _______________ or g(x) = _______________.

Reflections and Stretches of the function y = f(x)

Function	Transformation from y = f(x)	Mapping	Example
y = –f(x)	• a reflection in the x-axis	$(x,y)\rightarrow(x,-y)$	
y = f(–x)	• a reflection in the y-axis	$(x,y)\rightarrow(-x,y)$	
y = af(x)	• A vertical stretch about the x-axis by a factor of $\lvert a\rvert$. • If a < 0, then the graph is also reflected in the x-axis	$(x,y)\rightarrow(x,ay)$	
y = f(bx)	• A horizontal stretch about the y-axis by a factor of $\frac{1}{\lvert b\rvert}$. • If b < 0, then the graph is also reflected in the y-axis	$(x,y)\rightarrow(\frac{x}{b},y)$