Horizontal & Vertical Translations

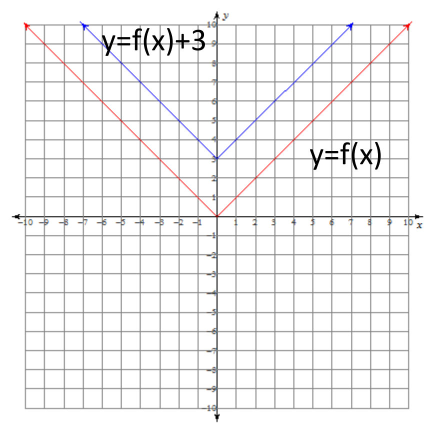
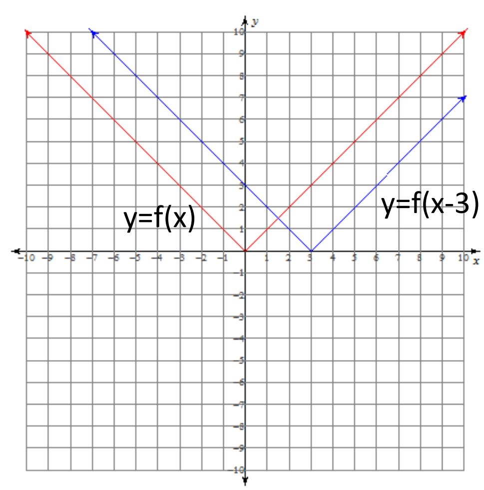
Transformation: *a change made to a figure or a relation such that the figure or the graph of the relation is shifted or changed in shape.*

Translation: *a transformation that shifts all points on the graph of a function up, down, left or right; a translation changes the location of the graph but not it shape or orientation.*

Mapping: *the relating of one set of points to another set of points so that each point in the original set corresponds to exactly one point in the image set. For example, the relationship between the coordinates of one set of points* (x, y) *and the coordinates of a corresponding set of points* $(x, y-4)$ *can be shown using the mapping notation* $(x, y) \rightarrow (x, y-4)$.

Image Point: *the point that is the result of a transformation of a point on the original graph.*

Translations of the function y = f(x)

Function	Transformation from y = f(x)	Mapping	Example
y – k = f(x) OR y = f(x) + k	• a vertical translation (a transformation on y) • If k > 0, the translation is up • If k < 0, the translation is down	$(x, y) \rightarrow (x, y+k)$	
y = f(x – h)	• a horizontal translation (a transformation on x) • If h > 0, the translation is to the right. • If h < 0, the translation is to the left.	$(x, y) \rightarrow (x+h, y)$	

Example 1: Graph Translations of the Form y – k = f(x) and y = f(x – h)

a. Graph the functions $y = x^2$, $y + 4 = x^2$, and $y = (x-6)^2$ on the same set of coordinates axes.

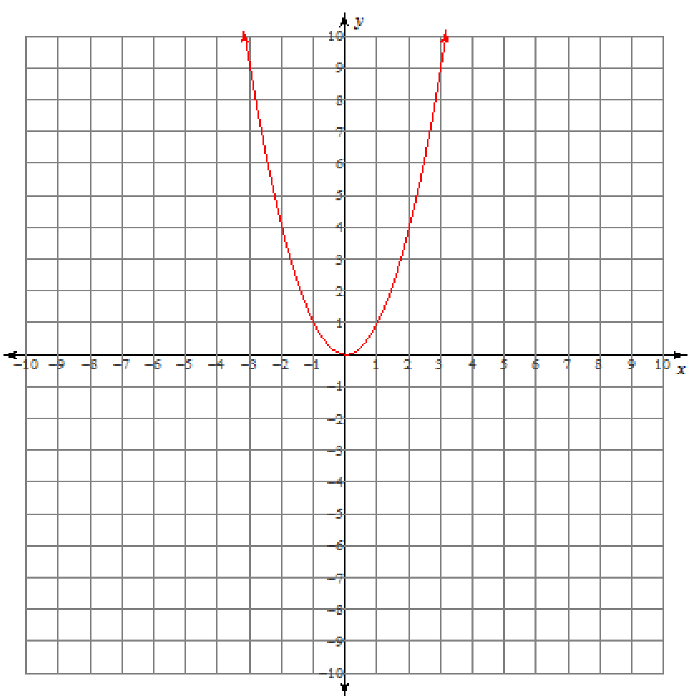
b. Describe how the graphs of $y + 4 = x^2$ and $y = (x-6)^2$ compare to the graph of $y = x^2$.

Solution:

a. To graph the functions complete the following tables.

Rearrange the equation $y + 4 = x^2$ to ____________________.

x	$y = x^2$		x	$y = x^2 - 4$		x	$y = (x-6)^2$
–3	9		–3				9
–2	4		–2				4
–1	1		–1				1
0	0		0				0
1	1		1				1
2	4		2				4
3	9		3				9



b. **Compare the graphs of $y = x^2$ and $y = x^2 - 4$.**

For $y = x^2 - 4$, the input values of x remain the same but the output values for y ______________________________.

Each point (x, y) on the graph $y = x^2$ is transformed to (x, y – 4). In mapping notation, ______________________________.

Thus the graph of $y + 4 = x^2$ is the graph of $y = x^2$ **translated vertically** __________________.

Compare the graphs of $y = x^2$ and $y = (x-6)^2$.

For $y = (x-6)^2$, to maintain the same output values as $y = x^2$, the input values for x ________________.

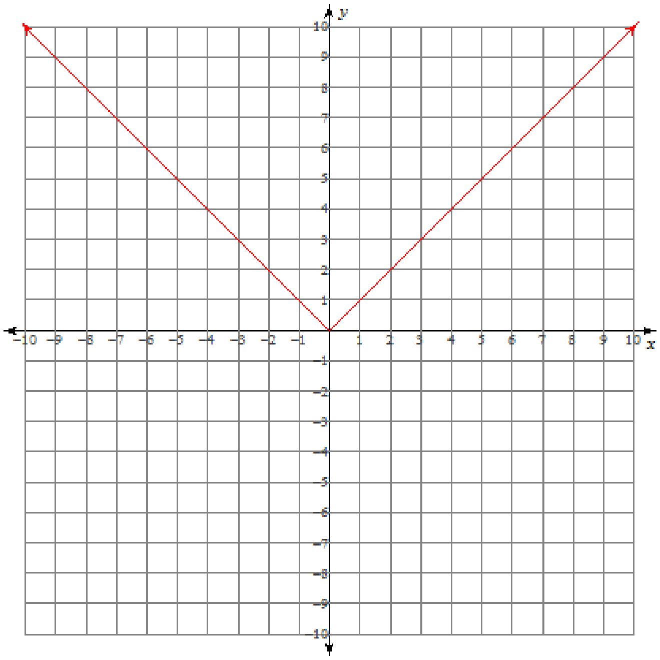
Each point (x,y) on the graph $y = x^2$ is transformed to (x + 6,y). In mapping notation, ________________.

Thus the graph of $y = (x-6)^2$ is the graph of $y = x^2$ **translated horizontally** ________________________.

Example 2: Horizontal & Vertical Translations

a. Sketch the graph of $y = |x+5| + 1$. State the domain and range of the function.

Solution: Start with the graph of the base function $y = |x|$.



For each point on the graph of $y = |x|$ apply a

horizontal translation of ___________________________

and a **vertical translation** of ______________________

to obtain the graph of $y = |x+5| + 1$.

Note that the point (0, 0) is transformed to become the point (_____, _____).

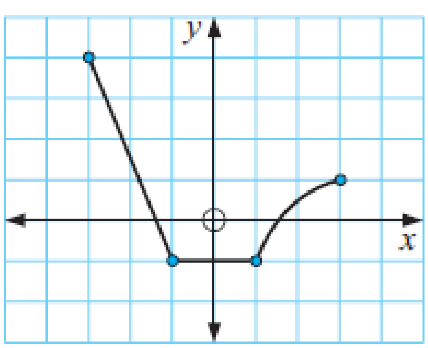
Mapping: $(x, y) \rightarrow$ __________________________

Domain: ___________________________________

Range: _____________________________________

b. The function $y = f(x)$ is illustrated below. Sketch the graph of $y = f(x-2) - 1$. State the domain and range of the function.

Solution:



For each point on the graph of $y = f(x)$ apply a

horizontal translation of _________________________

and a **vertical translation** of ______________________

to obtain the graph of $y = f(x-2) - 1$.

Mapping: $(x, y) \rightarrow$ __________________________

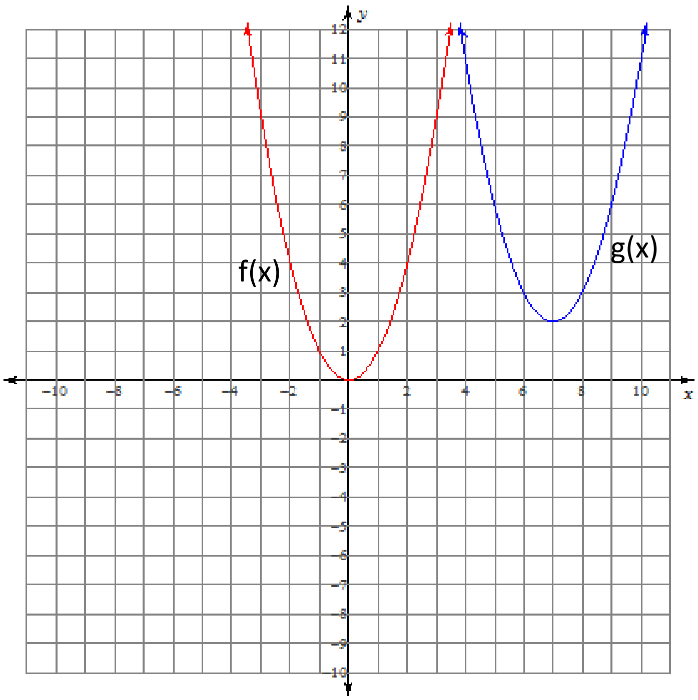
Domain: ___________________________________

Range: _____________________________________

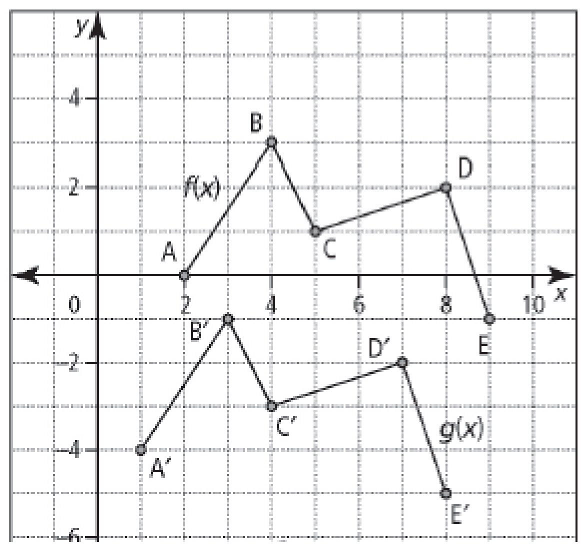
Example 3: Determine the Equation of a Translated Function

Describe the translation that has been applied to the graph of f(x) to obtain the graph of g(x). Determine the equation of the translated function in the form y – k = f(x – h).

a.



b.



Solution:

Choose key points on the graph of y = f(x) and locate the corresponding image points on the graph of g(x).

a.

$f(x) \rightarrow g(x)$

$(0,0) \rightarrow (7,2)$

$(-1,1) \rightarrow$

$(1,1) \rightarrow$

$(-2,4) \rightarrow$

$(2,4) \rightarrow$

$(x,y) \rightarrow$ (________, __________)

b.

$f(x) \rightarrow g(x)$

$(2,0) \rightarrow$

$(4,3) \rightarrow$

$(5,1) \rightarrow$

$(8,2) \rightarrow$

$(9,-1) \rightarrow$

$(x,y) \rightarrow$ (_______, ________)

Equation of the translated function:

____

Equation of the translated function:

____